2017-2018 S.6 畢業試 MC Solution

1. $8^{333} \cdot \left(\frac{1}{3^{999}}\right)$

$= (2^3)^{333} \cdot \left(\frac{1}{3^{999}}\right)$

$= \frac{2^{999}}{3^{999}}$

$= \left(\frac{2}{3}\right)^{999}$ $\boxed{C}$

2. $\frac{3a+2b}{3b} = 4 + \frac{2a}{b}$

$\frac{3a+2b}{3b} \times 3b = 4 \times 3b + \frac{2a}{b} \times 3b$

$3a+2b = 12b + 6a$

$-10b = 3a$

$b = -\frac{3a}{10}$ $\boxed{A}$

3. $4a^2 + ab - 3b^2 + 8a - 6b$

$= (4a-3b)(a+b) + 2(4a-3b)$

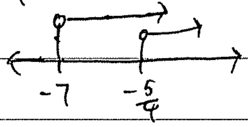
$= (4a-3b)[(a+b)+2]$

$= (4a-3b)(a+b+2)$ $\boxed{A}$

4. $4x+5>0$ 或 $x > \frac{2x-7}{3}$

$4x > -5$ $\quad$ $3x > 2x-7$

$x > -\frac{5}{4}$ $\quad$ $x > -7$



$\therefore x > -7$ $\boxed{C}$

5. $0.2675943$

$= 0.26759$ (準確至5位有效數字)

$\boxed{C}$

6. $f(x) = 3x^2 - 4x + 2k$

$f(3) = 0$

$3(3)^2 - 4(3) + 2k = 0$

$15 + 2k = 0$

$k = -\frac{15}{2}$

$\therefore f(x) = 3x^2 - 4x - 15$

$f\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^2 - 4\left(\frac{1}{3}\right) - 15$

$= \frac{1}{3} - \frac{4}{3} - 15$

$= -1 - 15$

$= -16$ $\boxed{A}$

7. 左方 $= mx^2 + 4mx + nx - 3n$

$= mx^2 + (4m+n)x - 3n$

右方 $= 3x^2 + mx + 2m + 21$

比較同類項:

$\therefore m = 3$ $\quad$ $4(3) + n = 3$

$n = -9$

$\boxed{A}$

8. $y=(bx-1)^2-b$

$y=b^2x^2-2bx+1-b$

$\because b^2>0$

$\therefore$ 開口向上, A, C ✗

$y=(bx-1)^2-b$

$=b^2x^2-2bx+1-b$

$=b^2(x^2-\frac{2}{b}x)+1-b$

$=b^2(x-\frac{1}{b})^2-b^2(\frac{1}{b})^2-b$

$=b^2(x-\frac{1}{b})^2-b$

頂點 $(\frac{1}{b},-b)$

$\because \frac{1}{b}=(+)$

$\therefore b=(+)$

$\therefore$ 頂點y坐標 $=-b=(-)$

$\boxed{D}$

9. $x^2+(x+5)k-16=0$

$x^2+kx+5k-16=0$

$\because$ 有等根

$\therefore \Delta=0$

$(k)^2-4(1)(5k-16)=0$

$k^2-20k+64=0$

$k=4$ 或 $16$

$\boxed{C}$

10.

售價 $=10000\times(1-25\%)\times(1+25\%)$

$=\$9375$

$\boxed{B}$

11. $3a=2b$ 及 $b:c=4:5$

$a:b=2:3$ 及 $b:c=4:5$

設 $a=8k$, $b=12k$, $c=15k$

| a | b | c |
|---|---|---|
| 2 | 3 | |
| | 4 | 5 |
| 8 | 12 | 15 |

$\therefore (2a+b):(b+2c)$

$=2(8k)+12k:[12k+2(15k)]$

$=28k:42k$

$=2:3$

$\boxed{A}$

12. $z=\frac{kx^2}{y}$

代 $x=y=z=1$

$\therefore k=1$

新的 $z=\frac{(1)(1-20\%)^2}{(1+25\%)}$

$=0.512$

$0.512-1=-0.488$

$\therefore$ 減少了 $48.8\%$

$\boxed{D}$

13. $T(1)=4$ $\xrightarrow{+6}$
$T(2)=10$ $\xrightarrow{+8}$
$T(3)=18$ $\xrightarrow{+10}$
$T(4)$ $\xrightarrow{+12}$
$T(5)=$ $\xrightarrow{+14}$
$T(6)=$

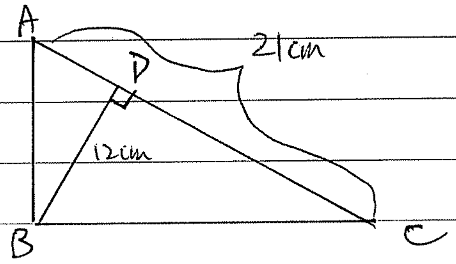
$\therefore\ T(6)=4+6+8+10+12+14$
$=54.$

$\boxed{B}$

14. $\dfrac{200-\boxed{\frac{0.5}{2}}}{10}$ 最大絕對誤差

$=19.975g.$

$\boxed{C}$

15. 


設 $AD=x$ cm
$DC=(21-x)$ cm.

$\frac{12x}{2}+66=\frac{12(21-x)}{2}$

$12x+132=252-12x$

$24x=120$

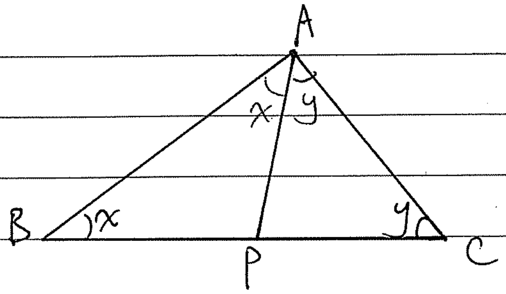
$x=5.$

$AD=5$ cm
$DC=16$cm.

$AB^2=12^2+5^2$
$AB=13$
$BC^2=12^2+16^2$
$BC=20$
$\therefore \triangle ABC$ 周界
$=13+20+21$
$=54$cm.

$\boxed{A}$

16.




$\because AP = PC$ $\because$ AP是中線

$\therefore \angle PAC = y$ $\therefore BP = PC = AP$

$\therefore \angle BAP = x$

$2x + 2y = 180^\circ$

$x + y = 90^\circ$ $\boxed{C}$

18. 半球体体積 $= \frac{1}{2}\left[\frac{4}{3}\pi r^3\right]$

$= \frac{2}{3}\pi r^3$

圓錐体体積 $= \frac{1}{3}\pi r^2 h$

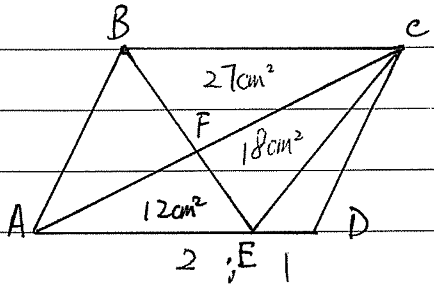
$\therefore \left[\frac{2}{3}\pi r^3\right] = \frac{1}{3}\pi r^2 h \times 2$

$r = h$

$\therefore 1:1$

$\boxed{D}$

17.




$AE : ED = 2:1$

$AE : BC = 2:3$

$\because \triangle BFC \sim \triangle EFA$

$\therefore \frac{\triangle BFC\text{面積}}{12} = \left(\frac{3}{2}\right)^2$

$\triangle BFC$面積 $= 27cm^2$

$AF : FC = 2:3$

$\therefore \triangle CEF$面積 $= 12 \times \frac{3}{2}$

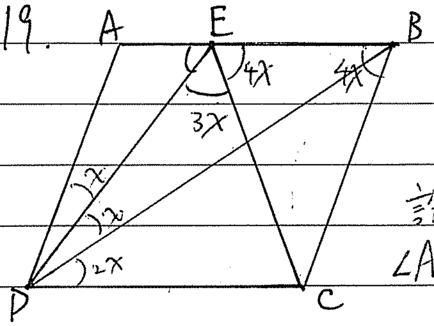
$= 18cm^2$

$\triangle CDE$面積 $+ 12 + 18 = 27 + 18$

$\triangle CDE$面積 $= 15cm^2$

$\boxed{A}$

19.




設 $\angle ADE$ 為 $x$

$\because$ ED 平分 $\angle ADB$

$\therefore \angle ADE = \angle BDE = x$

$\angle CDB = 2x$

$\because CD = CE$

$\therefore \angle CDE = \angle CED = 3x$

$\angle EBC = \angle ADC = 4x$

$\because CE = CB$

$\therefore \angle CEB = \angle EBC = 4x$

$\angle DAE = 180^\circ - 4x$ (同旁內角, AB//CD)

在 $\triangle ADE$:

$x + 180^\circ - 4x = 3x + 4x$ ($\triangle$外角)

$x = 18$

$\therefore \angle AED = 180^\circ - 7(18) = 54^\circ$ $\boxed{B}$

20. 設外角為 $x$

內角為 $8x$

$\therefore\ x+8x=180^\circ$

$x=20^\circ$

$\therefore$ 內角 $=160^\circ$ I ✓

$\frac{360^\circ}{20^\circ}=18$ $\therefore$ 有18條反射對稱軸

III ✓

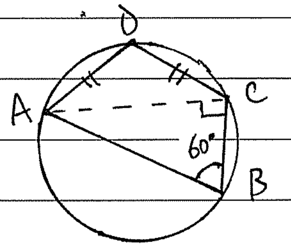
對角線數目 $=\frac{18\times15}{2}$

$=135$

$\therefore$ II ✗

$\boxed{C}$

21.



AB 為直徑

$\therefore\ \angle ACB=90^\circ$ (半圓上的圓周角)

$\angle ADB=180^\circ-60^\circ$ (圓內接四邊形對角)

$=120^\circ$

$\because\ \angle ACD=\angle CAD$ (等腰△底角)

$\therefore\ \angle ACD=\frac{180^\circ-120^\circ}{2}$ (△內角和)

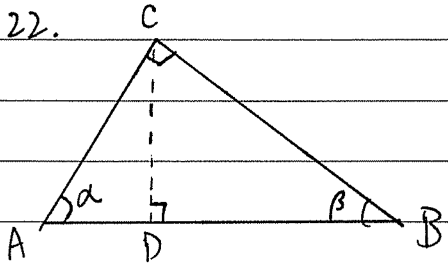
$=30^\circ$

$\therefore\ \angle DCB=90^\circ+30^\circ$

$=120^\circ$

$\boxed{B}$

22.



$\sin\alpha=\frac{BC}{AB}$

$\therefore\ AB=\frac{BC}{\sin\alpha}$

$\sin\beta=\frac{CD}{BC}$

$\therefore\ CD=BC\sin\beta$

$\therefore\ \frac{CD}{AB}=\frac{BC\sin\beta}{\frac{BC}{\sin\alpha}}$

$=\sin\alpha\cdot\sin\beta$

$\boxed{A}$

23. $hx+5y-10=0$

$m=-\frac{h}{5}$

$5x-ky+4=0$

$m=\frac{5}{k}$

$\because$ 互相垂直，

$\therefore\ -\frac{h}{5}\times\frac{5}{k}=-1$

$h=k$

代 $x=0$ 入 $hx+5y-10=0$

$5y-10=0$

$y=2$

代 $(0,2)$ 入 $5x-ky+4=0$

$-2k+4=0$

$k=2$

$\therefore\ h=2$ $\boxed{B}$

24. $(3, 240^\circ)$ 順時針 $90^\circ$ 旋轉

$\Rightarrow (3, 150^\circ)$

轉直角坐標

Shift − ,

$(3, 150^\circ)$

$= -2.598$

RCL ,

$= 1.5$

$D\left(-\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$

25. $L_1: ax - by + 1 = 0$

$m = \frac{a}{b} < 0$

$y$截距 $= \frac{1}{b} > 0$

$\therefore b > 0$. II ✗

$\therefore a < 0$. I ✓

$L_2: y = c$

$y$截距 $= c = \frac{1}{b}$

$\therefore b = \frac{1}{c}$ III ✓

$\therefore$ $\boxed{C}$

26. 圓: $x^2 + y^2 - 6x + ky - 32 = 0$

圓心 $= \left(3, \frac{-k}{2}\right)$

代 $\left(3, \frac{-k}{2}\right)$ 入 $3x + 2y = 8$

$3(3) + 2\left(\frac{-k}{2}\right) = 8$

$k = 1$

$\boxed{B}$

27. 圓: $2x^2 + 2y^2 - 16x + 24y - 31 = 0$

$x^2 + y^2 - 8x + 12y - \frac{31}{2} = 0$

圓心 $= (4, -6)$

代 $(4, -6)$ 入 $2x + y - 2$

$= 2(4) - 6 - 2$

$= 0$ I ✓

半徑 $= \sqrt{(4)^2 + (6)^2 - \left(-\frac{31}{2}\right)}$

$= \sqrt{\frac{135}{2}}$

$= 3\sqrt{\frac{15}{2}}$ II ✓

III 距離

$= \sqrt{(4-0)^2 + (-6-0)^2}$

$= \sqrt{52} < 3\sqrt{\frac{15}{2}}$

$\therefore$ 在 C 內 III ✓

$\boxed{D}$

28. 6★◇ 有 100 個 3 位數

可以被 6 整除有 17 個

$\therefore$ 概率 $= \frac{17}{100}$

$\boxed{B}$

29. 期望值

$= 20 \times \frac{5}{10} + 50 \times \frac{3}{10} + 100 \times \frac{2}{10}$

$= 10 + 15 + 20$

$= 45$ $\boxed{D}$

30. $\{y-4, y-1, y, y+3\}$

平均值 $= \dfrac{y-4+y-1+y+y+3}{4}$

$= \dfrac{4y-2}{4} = y-\frac{1}{2}$

中位數 $= \dfrac{y-1+y}{2} = y-\frac{1}{2}$

分佈域 $= (y+3)-(y-4)$

$= 7.$

$\{y-3, y-2, y+1, y+2\}$

平均值 $= \dfrac{y-3+y-2+y+1+y+2}{4}$

$= \dfrac{4y-2}{4} = y-\frac{1}{2}$

中位數 $= \dfrac{y-2+y+1}{2} = y-\frac{1}{2}$

分佈域 $= (y+2)-(y-3)$

$= 5$

$\therefore$ I, II ✓ III ✗

$\boxed{A}$

乙 31. $2x^3y^9$

$x^4y^7z^3$

第三個數式

HCF $= x^2y^3$ $\Rightarrow$ 一定是 $x^2y^3$

LCM $= 6x^4y^9z^5$

$\therefore$ $6x^2y^3z^5$.

$\boxed{D}$

32. $CD00AE0000000_{16}$

$= 12\times16^{12}+13\times16^{11}+10\times16^{8}$
$+14\times16^{7}$

$= (12\times16+13)\times16^{11}+(10\times16+14)\times16^{7}$

$= 205\times16^{11}+174\times16^{7}$

$\boxed{A}$

33. $y = ab^x$

$\log_5 y = \log_5 ab^x$

$\log_5 y = \log_5 a + x\log_5 b$

$\therefore$ $y$ 截距 $= -1$.

$\log_5 a = -1$

$a = 5^{-1}$

$a = \frac{1}{5}$

$\boxed{C}$

34. $x^2+ax+b=0.$

兩根之和 $= -a$.

$(-1+4i)+(-1-4i) = -a$.

$-2 = -a$

$a = 2.$

兩根之積 $= b$

$(-1+4i)(-1-4i) = b.$

$(-1)^2-(4i)^2 = b$

$1+16 = b$

$b = 17$

$a+b = 17+2$

$= 19.$ $\boxed{D}$

35. $2x-5y\le 1$

考慮方程：

$2x-5y=1$

$y=\frac{2}{5}x-\frac{1}{5}$

∴ 斜率 $=\frac{2}{5}$ A,B ✗

y截距 $=-\frac{1}{5}$

代 (0,0) 入不等式

左 $=2(0)-5(0)$

$=0$

$\le 1=$ 右

∴ $\boxed{D}$

36. $a_5=768$

$a_9=48$

$ar^4=768$

$ar^8=48$

$\frac{1}{r^4}=16$

$r^4=\frac{1}{16}$

∴ $r=\frac{1}{2}$ 或 $r=-\frac{1}{2}$, $a=12288$

不是整數. I ✗

$\frac{\log a_{13}}{\log a_9}=\frac{\log 3}{\log 48}$

$=0.284$

$<\frac{1}{2}$ II ✓

$a_1+a_3+\dots+a_{2n-1}$

$=\frac{a}{1-\frac{1}{4}}=\frac{12288}{\frac{3}{4}}=16384$

$>16383$ III ✗

$\boxed{B}$

37. 最高值與最低值是3和-3.

∴ 倍大了3倍.

週期是 $180°$

∴ A,B ✗

$\cos(90°-2x)=\sin 2x$

∴ $\boxed{C}$

38. $2\cos\theta-3\tan\theta=0$

$2\cos\theta-\frac{3\sin\theta}{\cos\theta}=0$

$2\cos^2\theta-3\sin\theta=0$

$2(1-\sin^2\theta)-3\sin\theta=0$

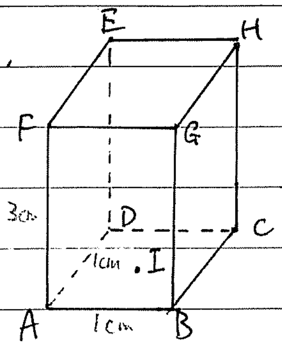
$-2\sin^2\theta-3\sin\theta+2=0$

$\sin\theta=-2$ 或 $0.5$
(捨去)

$\theta=30°$ 或 $150°$

∴ $\boxed{A}$

39.




AD = 1cm

設 AB = 1cm

AF = 3cm

$AC^2=1^2+1^2$

$AC=\sqrt{2}$

$AI=\frac{\sqrt{2}}{2}$

$FI^2=3^2+\left(\frac{\sqrt{2}}{2}\right)^2$

$FI=\sqrt{\frac{19}{2}}$

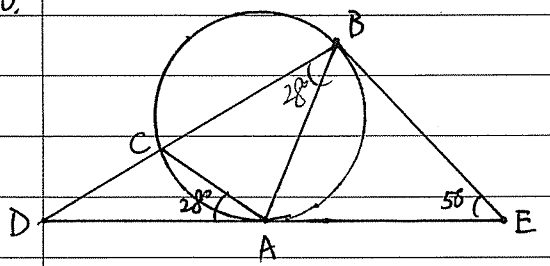
$\cos\theta=\frac{\frac{\sqrt{2}}{2}}{\sqrt{\frac{19}{2}}}$

$\theta=76.7°$

$\approx 77°$

$\boxed{A}$

40.



$\angle ABC = 28^\circ$ (交錯弓形的圓周角)

$\angle EAB = \angle EBA$ (由外點引切線)

$\therefore\ 2\angle EBA + 50^\circ = 180^\circ$ (△內角和)

$\angle EBA = 65^\circ$

$\angle CDA + 50^\circ + (28^\circ + 65^\circ) = 180^\circ$

(△內角和)

$\angle CDA = 37^\circ$ $\boxed{A}$

41. 代 $(0,-2)$ 及 $(1,-4)$ 入 $\begin{cases} 5x+3y-1=0 & \text{——(1)} \\ x^2+y^2+2x-4y-11=0 & \text{——(2)} \end{cases}$

代 $(0,-2)$:

$5(0)+3(-2)-1 = -7 \neq 0$ ✗

代 $(1,-4)$:

$5(1)+3(-4)-1 = -8 \neq 0$ ✗

I ✗

半徑 $= \sqrt{\left(\frac{-2}{2}\right)^2 + \left(\frac{4}{2}\right)^2 - (-11)}$

$= \sqrt{16}$

$= 4$

直徑 $= 8$ II ✓

圓心 $(-1, 2)$

代 $(-1,2)$ 入 $5x+3y-1=0$

左 $= 5(-1)+3(2)-1$

$= 0$

$=$ 右 III ✓

$\therefore$ $\boxed{D}$

42. P(射中至少2次)

$= P(2次) + P(3次)$

$= 3\left(\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}\right) + \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$

$= \frac{20}{27}$

$\boxed{B}$

43. $(C^{12}_3 - C^{10}_1)\times 3!$

$= 1260$

$\boxed{C}$

44. 新的平均分 $= 1.5\mu$

A, B ✗

全班增加分數不會影響小華

小華的標準分 $= z$

$\boxed{D}$

45. 方差 $= 100$

標準差 $= \sqrt{100}$

$= 10$

新的標準差 $= \frac{10}{2}$

$= 5$

$\therefore$ 新的方差 $= 5^2$

$= 25$

$\boxed{B}$